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a.roche@larcroa.fr**Keywords**painting defect, mechanical degradation, stress concentration factor ( $Kt$ ), finite element analysis, mechanical fatigue, endurance limit**Abstract**

Within our research on the mechanical behavior of paintings, the results obtained from static tests did not allow us to explain certain phenomena connected to failure and damage of the paint layers submitted to mechanical fatigue. The fragility of paint layers is mainly due to the presence of morphological defects, which provoke stress concentration zones characterized by a specific stress concentration factor ( $Kt$ ). The aim of this study is to propose a simplified mathematical expression to calculate the endurance limit for paint films. Using finite element analysis (FEA), a set of virtual samples, built using cross-section samples from real paintings, were analyzed. In order to validate the  $Kt$  value, tests were repeated under different environmental conditions. Based on the mechanical modeling of the cross-sections, the method allowed us to test the paint materials virtually and to deepen our understanding of specific mechanical phenomena connected to failure and the development of cracks. By extrapolating the data from 85 simulations, a simplified equation for the determination of the endurance limit of a paint layer under mechanical fatigue allowed the fragility and the mechanical degradation risks of a painting to be evaluated.

**INTRODUCTION**

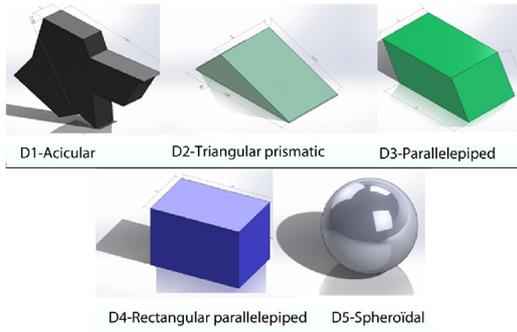
The presence of a network of cracks on the surface of a painting is a common sight. The occurrence of these alterations is due to the combined effect of various degradation factors. Mecklenburg (2007) has shown us that environmental fluctuations are one of the predominant factors in this process, constantly impacting on artworks. Vibrations are also another factor though more occasional. They both induce mechanical fatigue, which initiates cracking in the paint layers. In order to understand these mechanisms, this research focused mainly on the work of Griffith (1921), Irwin (1957), Wöhler (1867), and others who have provided solutions to structural support problems, such as metal bridges or building frameworks. No literature was found on the endurance limit for paint layers, although research has been conducted on the mechanical degradation phenomena of paints on wooden panels (Rachwał et al. 2012, Bratasz and Reza Vaziri Sereshk 2018).

The aim of this study is to propose a simple equation to calculate the endurance limit for a paint film that takes into consideration its ultimate strength and microstructure. This mechanical value is necessary to evaluate degradation risks on paintings due to mechanical fatigue. To solve this problem, finite element analysis (FEA), which combines mathematics with mechanical and numerical analysis to create a new science, was used. Advances in information technologies, increased computing power, and improvements to interface interactivity have enabled wider dissemination of this method in engineering offices and laboratory work. FEA was used to assess the stress concentration factor and to characterize the mechanical fatigue endurance limit for a paint film.

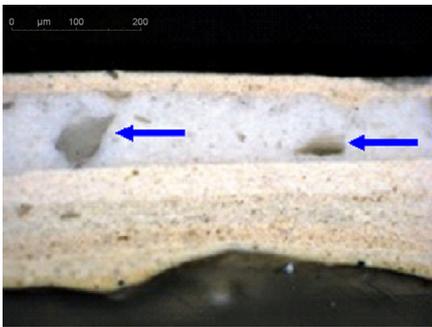
**MICROSTRUCTURE OF A PAINTING**

Paint films are heterogeneous materials. They are composed of a polymeric matrix and natural or synthetic pigments of different sizes whose shape may be acicular, prismatic, lamellar, or spheroidal.

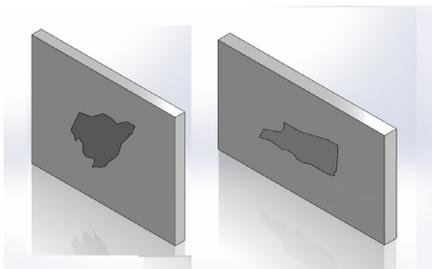
Defects are defined as acicular particles, crystals, or agglomerates whose dimensions are at least equal to or larger than one-third of the thickness of the layer. Starting with observations of 800 microscopic cross-sections of paint, the range of defects was reduced to five shapes: acicular (D1), triangular prismatic (D2), parallelepiped (D3), rectangular parallelepiped (D4), and spheroidal (D5) (Figure 1).



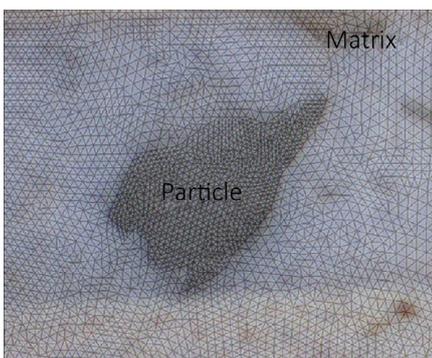
**Figure 1.** Defect shapes



**Figure 2.** Digital photography of a microscopic cross-section with defects



**Figure 3.** Transformation into a numerical model



**Figure 4.** Numerical meshing of a defect

Natural and synthetic polymeric films are much more fragile than solid particles. A painting's sensitivity to mechanical fatigue is due to its heterogeneity and the quantity and size of its previously defined defects.

## PROPOSED CALCULATION OF THE ENDURANCE LIMIT FOR A PAINT LAYER: $\sigma DP$

The fragility of a paint layer is strongly influenced by the presence of one or many defects. About 800 microscopic cross-sections of paint layers on paintings from the 17th century to the present day were observed in order to characterize any defects. The vast majority of the microscopic cross-sections<sup>1</sup> were found to have defects, showing that the probability of their presence in a paint layer was statistically high. These defects provoke stress concentration zones characterized by a stress concentration factor ( $Kt$ ).<sup>2</sup> In static mechanics, the breaking strength resistance of a material is characterized by its breaking stress ( $\sigma_r$ )<sup>3</sup> and in mechanical fatigue by its endurance limit ( $\sigma D$ ). To assess the resistance of a paint layer to mechanical fatigue, the endurance limit is related to both  $\sigma_r$  and  $Kt$ .

The endurance limit for paint film ( $\sigma DP$ ) can therefore be expressed mathematically as the ratio between  $\sigma_r$  and  $Kt$ .

$$\sigma DP = \frac{\sigma_r}{Kt} \quad \text{Equation 1}$$

## METHODOLOGY

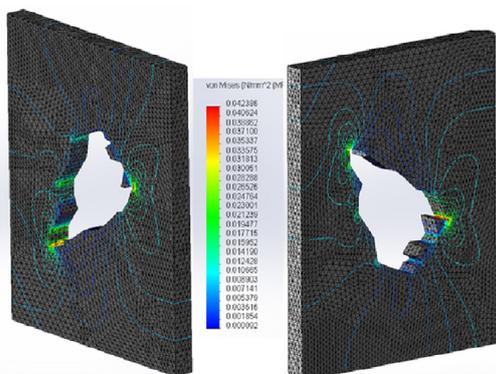
The idea was to take a digital photograph of a microscopic cross-section of a painting to determine the volume around a defect and to transform it into a digital sample so that FEA could be used to calculate the stress concentration factor. The process involved three steps.

Step 1: Selection of a photograph of a microscopic cross-section of a painting and identification of the defects. In this example, two defects were detected in a white layer. They were translucent, acicular in shape, and their size was superior to two-thirds of the layer thickness (Figure 2).

Step 2: Virtual cutting of the part of the microscopic cross-section containing the defect and the polymer matrix. After extraction, this zone was modeled in 3D with vector drawing software to transform the detail of the image into a numerical model (Figure 3).

Step 3: Meshing. FEA is based on the principle of subdividing a model into finite geometric-shaped elements interconnected by nodes. In this case, it was a standard volumetric mesh with tetrahedral elements. To get good mechanical resolution, the mesh of the defect must be finer than the polymer matrix mesh. When the numerical model was finished and meshed, it was configured (Figure 4).

Step 4: This step required knowledge of the mechanical properties of the materials. The mechanical values of painting materials collected by the LARCROA laboratory were used. To measure the elasticity modulus, elasticity threshold, breaking stress, and elongation at breaking point, the same procedure was applied.<sup>4</sup>



**Figure 5.** Three-dimensional plot with localization of stresses

The tests were undertaken on a universal testing machine (UTM). The characteristics of oil paint were obtained and a ceramic-like material was used as a reference for the particles. These parameters were fed into the FEA. Once the data had been entered into the software,<sup>5</sup> the digital modeling created a virtual sample whose behavior was considered similar to a real paint layer.

In order to apply the correct values for displacements and loads to this virtual sample, it had to be put into a real context. Loads were calculated according to the dimensions of a real painting and the tension was calculated according to statistical values (Cappriotti and Iaccarino Idelson 2004). The application of the loads relied on the geometry of the sample. Following calculations by the software, the stresses were visible on a 3D plot (Figure 5). On this plot, the stresses were represented by color lines corresponding to values. These plots were very detailed, precisely locating the stress concentration zones. In the case of the acicular defect, two high-stress zones were visible. It was here that mechanical degradation was most likely to occur. As the plot shows the minimum and maximum stresses,  $Kt$  can be calculated according to the ratio of maximum stress to normal stress. In this case, normal stress is mostly equal to minimum stress and therefore can be expressed thus:

$$Kt = \frac{\sigma_{max}}{\sigma_{mini}} \quad \text{Equation 2}$$

To validate the constant value for  $Kt$ , a high number of stress concentration factors had to be calculated using two different methods. In the first,  $Kt$  values starting at 50 digital photos of microscopic cross-section paint layers were calculated by applying the methodology mentioned earlier. In the second,  $Kt$  resulted from the digital models obtained by combining the five forms of particles with the seven matrices of the different polymers.

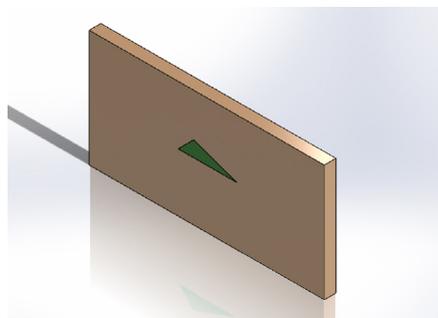
### **Kt ASSESSMENT STARTING WITH DEFECT MODELING ON 50 MICROSCOPIC CROSS-SECTIONS FROM OIL PAINTINGS**

In this part of the study,  $Kt$  was calculated using the defect modeling protocol on 50 microscopic cross-sections from oil paintings by artists including Le Brun, Delobel, Courbet, Gleizes, Picasso, Chagall, Derain, Braque, and Foujita. In this range of 50 digital samples, defects were categorized according to the five defined shapes: D1, D2, D3, D4, and D5. The results are shown in Table 1.

**Table 1.** Calculation of percentage and  $Kt$  average according to the defect shapes in 50 digital samples

Shape	D1	D2	D3	D4	D5
Quantity %	40	27	17	6	10
<b>Average <math>Kt</math> (1)</b>	<b>115</b>	<b>98</b>	<b>74</b>	<b>65</b>	<b>38</b>

For each defect shape, the percentage and  $Kt$  average were calculated. The average value of  $Kt$  varied from 38 to 115. Acicular (D1) and triangular prismatic (D2) defects were the most frequent in the paintings (40% and 27%). Due to the presence of acute angles, they had the highest average  $Kt$  values (115 and 98) and constituted 67% of samples.



**Figure 6.** Example of a digital model with a polymeric matrix and a triangular prismatic defect (D2)

## **Kt** ASSESSMENT STARTING FROM 35 DIGITAL SAMPLES OF VARIOUS PAINTS

In the second step, 35 digital samples were produced resulting in the association of the five defect shapes with the seven matrices. The design of this new digital model allowed the  $Kt$  for the different types of paints to be calculated (Figure 6).

The polymeric matrices chosen were industrial paints used by artists and fine art paint:

1. Urethane alkyd paint—modified alkyd with urethane isocyanate
2. Ultra-resist Valenite—glycerophthalic paint
3. Astrad Elégance—alkyd emulsion
4. Liquitex professional acrylic
5. Lascaux acrylic colors
6. Lascaux gouache paint
7. Lefranc & Bourgeois Flashe vinyl emulsion

The mechanical properties of each paint were characterized in the laboratory according to tensile testing standards.

The previously described protocol steps—meshing, entry of mechanical data, application of displacements and loads, 3D plots, and  $Kt$  value calculation—were applied to the 35 digital samples, and the results can be seen in Table 2.

It should be noted that, for the same shape, the  $Kt$  values varied according to the polymer type of the matrix. Average  $Kt$  values for each defect shape varied from 15 to 112, with D1 and D2 having the highest (112 and 81). The values for each defect shape was significant. They can be seen in Tables 1, 2, and 3.

**Table 2.** Calculation of the  $Kt$  value according to the defect shapes in 35 digital samples

Paints/shapes	D1	D2	D3	D4	D5
Urethane alkyd paint	116	86	26	22	15
Glycerophthalic paint	116	90	30	22	15
Alkyd emulsion	114	97	31	22	16
Liquitex® professional acrylic	115	86	30	22	15
Lascaux® acrylic	115	80	29	21	15
Lascaux® gouache	94	56	23	18	12
Flashe® vinyl emulsion	113	73	28	21	14
<b>Average <math>Kt</math> (2)</b>	<b>112</b>	<b>81</b>	<b>24</b>	<b>21</b>	<b>15</b>

**Table 3.** Average  $Kt$  (1) and  $Kt$  (2) values for each defect shape

Shape	D1	D2	D3	D4	D5
<b>Average <math>Kt</math> (1)</b>	<b>115</b>	<b>98</b>	<b>74</b>	<b>65</b>	<b>38</b>
<b>Average <math>Kt</math> (2)</b>	<b>112</b>	<b>81</b>	<b>24</b>	<b>21</b>	<b>15</b>

Since D1 and D2 defects have the highest  $Kt$  values and, as previously stated, are the most common in paint layers, they therefore have the most significant effect on the mechanical resistance of the paint film. If a paint

layer is found to have just one defect of this type, it may dramatically reduce its resistance.

Plotting  $\sigma DP$  as a function of  $Kt$  with Equation 1, for a constant ultimate stress value, the endurance limit for a paint film decreases rapidly from  $Kt=20$  to  $Kt=80$  for D5, D4, and D3 defects and then stabilizes between 80 and 160 for D2 and D1 defects. Six  $Kt$  values (80, 96, 100, 105, 120, and 140) from this last zone were then chosen for testing (Figure 7).

## CONSERVATION CONDITIONS REQUIRED FOR A STRETCHED PAINTING

The conservation measures recommended for canvas paintings:

1. recommended environmental values according to CCI Note 10/4 (Arnold and McKay 2016): relative humidity (RH) =  $55 \pm 5\%$  and  $\Delta T = 18^\circ \pm 2^\circ\text{C}$ ;<sup>6</sup>
2. the mechanical condition required according to Roche (2016, 99), that is, an endurance limit higher than or equal to the maximum stress variation of the painting<sup>7</sup>:

$$\sigma DP = \frac{\sigma_r}{Kt} \geq \Delta\sigma_{paint} \quad \text{Equation 3}$$

3. the painting's state of tension at 55% RH and 20°C, which must be<sup>8</sup>:

$$15 \text{ daN/m} < \text{tension} < 20 \text{ daN/m} \quad \text{Equation 4}$$

Knowing that tension or stresses vary in a painting as a function of humidity and temperature, the last condition is very variable. If stress or tension variations are inferior or equal to the endurance limit, the risks of mechanical degradation are very low and a painting's integrity is not in danger. If stress or tension variations are equal or slightly above the limit, risks of mechanical degradation are limited. However, if stress or tension variations are much greater than the endurance limit, risks of mechanical degradation and to a painting's integrity may be significant.

## STRESS CONCENTRATION FACTOR ( $Kt$ ) AS A CONSTANT VALUE

The hypothesis of using a constant value for  $Kt$  was validated by testing a set of 85 digital samples in two different climatic scenarios:

- standard conservation conditions: RH of 50%–60% and a constant temperature;
- drier conditions: RH of 30%–60% and a constant temperature.

Taking the maximum and minimum hygrometric value, it was possible to calculate the  $\Delta\sigma_1$  stress or tension variations at RH = 50% and RH = 60%, and the  $\Delta\sigma_2$  stress or tension variations at RH = 30% and RH = 60% from fourth-degree polynomials (Roche 2003, 99) for each digital paint tested.

$$\sigma(RH) = a + bRH + cRH^2 + dRH^3 + eRH^4 \quad \text{Equation 5}$$

The calculation of the endurance limit (Equation 1) was carried out for each digital sample at six  $Kt$  values (80, 96, 100, 105, 120, and 140) and according to the ultimate stress value for the series of 85 paints tested (oil

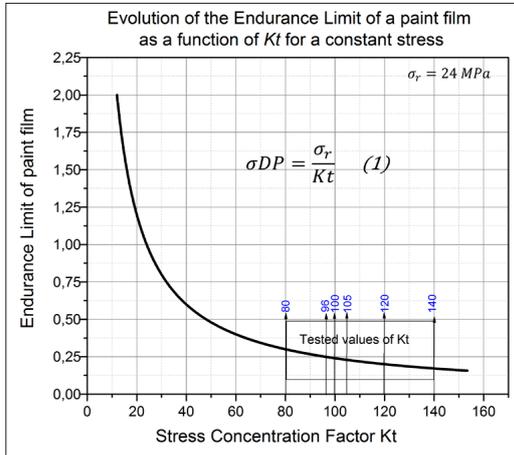


Figure 7. Evolution of the endurance limit to find  $Kt$  testing values

paintings, industrial paintings, and fine-art paintings). The endurance limit values ( $\sigma DP$ ) for each sample were compared with the stress variation values ( $\Delta\sigma_1$  and  $\Delta\sigma_2$ )<sup>9</sup> using the following criteria:

- $\sigma DP > \Delta\sigma_{1,2}$  (Equation 6) = no mechanical risks to the paint layers;
- $\sigma DP \leq \Delta\sigma_{1,2}$  (Equation 7) = limited or significant mechanical risks to the paint layers.

For each of the above criteria, the number of corresponding samples was expressed as a percentage of the total number. The results are shown in Table 4: in the first column, the  $\sigma DP$  of the paint film for the 85 samples was calculated at six  $Kt$  values. In columns 2 and 4,  $\sigma DP$  is greater than  $\Delta\sigma_1$  and  $\Delta\sigma_2$ , meaning mechanical conditions were respected in a high percentage of cases under standard or dry environmental conditions. In columns 3 and 5,  $\sigma DP$  is lower than  $\Delta\sigma_1$  and  $\Delta\sigma_2$ , meaning the mechanical conditions were not respected under standard or dry environmental conditions.

**Table 4.** Calculation of the percentage for each  $Kt$  value according to two different climatic scenarios and four criteria

1	50% > RH > 60%		30% > RH > 60%	
	2	3	4	5
	$\sigma DP > \Delta\sigma_1$	$\sigma DP \leq \Delta\sigma_1$	$\sigma DP > \Delta\sigma_2$	$\sigma DP \leq \Delta\sigma_2$
$Kt = 80$	100%	0%	46%	54%
$Kt = 96$	100%	0%	41%	59%
$Kt = 100$	100%	0%	39%	61%
$Kt = 105$	90%	10%	36%	64%
$Kt = 120$	81%	19%	35%	65%
$Kt = 140$	75%	25%	32%	68%

### VALIDATION OF A CONSTANT VALUE FOR $Kt$

In order to simplify the mathematical equation for the endurance limit, it became clear that  $Kt$  should be considered as a constant value, in the following terms:

1.  $Kt$  corresponds to the highest endurance limit percentage when it is greater than the stress variations ( $\Delta\sigma_1$ ) under standard conservation conditions.
2.  $Kt$  corresponds to the lowest endurance limit percentage when it is greater than the stress variations ( $\Delta\sigma_2$ ) under dry conservation conditions.

By applying these conditions, most  $Kt$  values could be discarded.

Under the first conditions, the tests showed that when  $\sigma DP > \Delta\sigma_1$ , mechanical conditions were respected in 100% of cases at  $Kt = 80, 96, 100$ , but at  $Kt = 105$ , the percentage was lower than 100%. The  $Kt$  values of 105, 120, and 140 could therefore be ruled out.

Under the second conditions, the tests showed that when  $\sigma DP > \Delta\sigma_2$ , the percentage of cases was lower at  $Kt = 80$  and  $96$  than at  $Kt = 100$  (39%). Therefore,  $Kt$  values of 80 and 96 could be ruled out.

The remaining value which best meets the requirements of conservation is  $Kt = 100$ .

By maintaining  $Kt$  at this value, the mathematical equation could be simplified and the endurance limit of the paint layer become proportional to the breaking stress (Equation 8):

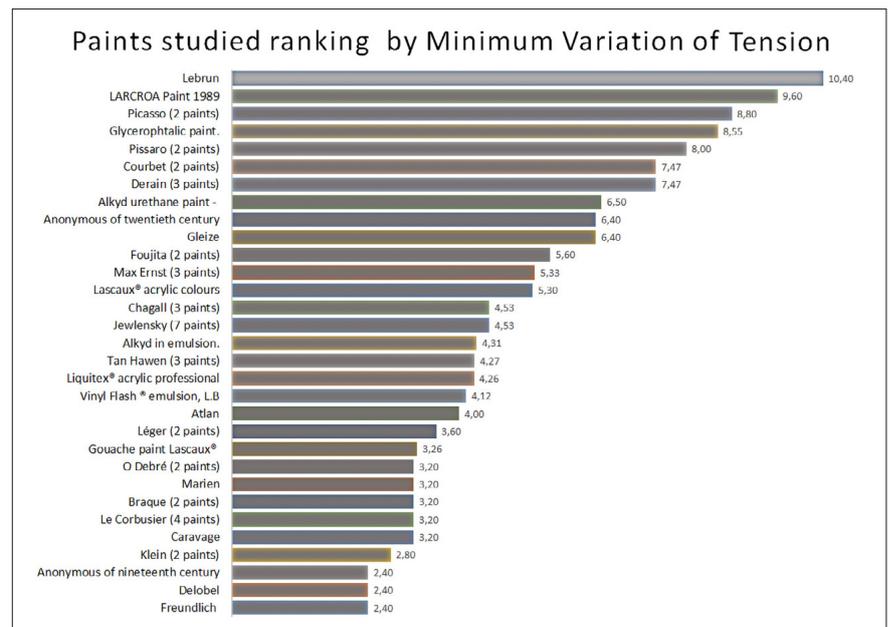
$$\sigma DP = \frac{\sigma_r}{100} \quad \text{Equation 8}$$

### HOW CAN THE ENDURANCE LIMIT OF A PAINT LAYER BE USED?

The simplified equation for the endurance limit of a paint layer in terms of mechanical fatigue allows the minimum tension variation ( $V_{\text{mini}}t$ )<sup>10</sup> of a painting to be calculated before incurring mechanical degradation. This property is expressed as the product of the endurance limit multiplied by the thickness of the paint layer ( $e$ ).

$$\sigma DP \times e = V_{\text{mini}}t \quad \text{Equation 9}$$

This property provides information about the sensitivity of a painting submitted to tension and/or environmental variations. The lower the endurance limit values, the greater a painting's fragility, allowing categorization accordingly. To illustrate this point, paintings have been ranked in Figure 8 in decreasing order of  $V_{\text{mini}}t$ .



**Figure 8.** Paint ranking bar chart

This fragility index for paintings could be used as a decision-making tool in preventive conservation. It provides collection managers with objective data to support the conservation of the paintings for which they are responsible. For example, the loan of a painting could be refused if the endurance limit value was too low, implying too high a risk of mechanical degradation. Collection managers could also provide detailed instructions for transportation or advice on display conditions.

By comparing the tension values obtained from the polynomial function of a type of painting (Equation 5) and its minimum tension variation, followed by the use of a suitable calculation module, it was possible to

assess the climatic impacts on the mechanical degradation of a painting by calculating the risk index.

## CONCLUSION

Stretched paintings are constantly submitted to tension and stress variations. This phenomenon leads to mechanical fatigue of the paint layer. Until now, a precise evaluation of the consequences of this fatigue has not been possible because the effects caused by structural damage to paint layers must be calculated using the endurance limit ( $\sigma_{DP}$ ).

For that reason, a mathematical equation was developed to calculate the endurance limit of a paint layer based on breaking stress, which characterizes the maximum resistance of a paint film, and the stress concentration factor due to the presence of defects.

To establish a constant value for  $Kt$ , FEA was used with digital models based on microscopic cross-sections of paintings. This enabled the constant value of  $Kt$  to be established as equal to 100. This would seem the most appropriate value for the conservation conditions of a painting in any climatic environment.

Given the complexity of painting structures and the variety of mechanical parameters used to calculate  $Kt$ , this approach can provide an interesting method to assess paint degradation risks. It also opens new avenues for developing preventive conservation methods that predict the impact of a climatic or physical environment.

## ACKNOWLEDGMENTS

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## NOTES

- <sup>1</sup> Approximate dimensions for the cross-sections: 300–450  $\mu\text{m}$ .
- <sup>2</sup>  $Kt$  (stress concentration factor): not expressed in units.
- <sup>4</sup> ISO 527-3 (ISO 2018) and ASTM D 882 (ASTM International 2018) standards.
- <sup>5</sup> SolidWorks Premium 2013, Simulation 2008, 2013X64 SPO. PCGLSS © 1992–2010 Computational Applications and System Integration, Inc., Dassault Systèmes.
- <sup>6</sup> In CCI Note 10/4 (Arnold and McKay 2016), the average humidity rate is 50% RH for a comfortable temperature of 20°C. For energy-saving reasons, an average humidity rate of 55% RH, regulation of  $\pm 5\%$  RH, and an average temperature of 18°C were used.
- <sup>7</sup> Estimation of the endurance limit of the characteristics of tensile tests.
- <sup>8</sup> “La pratica professionale ha poi permesso di arricchire la casistica dei dipinti montati a tensionamento noto tra 2002 e 2003 su circa venti dipinti di varie dimensioni, forme e pesi, no è mai stato necessario di oltrepassare la tensione di 2 N/cm . . .” Cappriotti, G and A. Iccarino Idelson, op. cit., p. 64.
- <sup>9</sup>  $\Delta\sigma_n = \sigma_x - \sigma_y$  – stress variation: expressed in MPa.
- <sup>10</sup>  $V_{\text{mini}}t$ : expressed in daN/m.

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